Master Degree in Mathematics and Mathematical Engineering

Mathematical Fluid Dynamics

6 ECTS

Academic Year 2019/20

Program

The equations of fluid mechanics

Continuous description of a fluid. The continuous medium assumption.
Lagrangian and Eulerian coordinates.
The transport theorem.
Mass Conservation Equation.
Linear Momentum Equation.
Cauchy's stress theorem.
Angular Momentum equation.
Energy Balance – First Law of Thermodyanmics.
Fluids: assumptions for a continuous medium to be a fluid.
Ideal Fluids: Euler fluids, Perfect ideal fluids.
Specific entropy and isoentropic fluids.
Viscous Fluids: deviation of the stress tensor.
Incompressible fluids: the incompressibility assumption.

Scaling and Dimensional Analysis

Dimensional analysis: Reynolds number, Mach Number, Frohde number.
A formal derivation of the relationship between compressible and incompressible fluids.

Setting up problems in fluid dynamics

Initial conditions.
Domain and Boundary condition.
Definition of well posed solutions.
Definition of classical solutions.
Definition of weak solutions.

Ideal Fluids: properties

Conservation of the total energy.
Bernoulli's Theorem and applications.
Kelvin's circulation Theorem and applications.

Vorticity

The vorticity equation in 2D and 3D.
The streamfunction for 2D flows.
Some examples of exact solutions for the 2D vorticity equation.
Some examples of exact solutions for the 3D vorticity equation.
Beltrami flows.

Relation between ideal and viscous fluids

D'Alambert Paradox.
Ideal fluids as an approximation of viscous fluids for vanishing viscous coefficient.
Boundary layers.

Modeling with Fluid Dynamics

Mixture of fluids. Combustion equations.
Hydrodynamical model for a gaseous star.
Mathematical modeling of Hemodyanmics. Fluid structure interaction. A 1D model for the flow inside an artery.

Incompressible fluids

Definition of classical solutions.
Locale existence theorem of classical solutions (without proof).
Breakdown of classical solutions: the Beale Kato Maida Theorem.

Incompressible Navier Stokes equations

Weak formulations of the incompressible Navier Stokes equations.
Leray's existence theorem for weak solutions for incompressible Navier Stokes equations.
Existence of the pressure.
Energy inequality.
Uniqueness of solutions for n=2
The problem of uniqueness of solutions for n=3.
Serrin's uniqueness condition.

Compressible Euler equations

Existence of local classical solutions.
Symmetrazibility of the compressible Euler system.